Invest-As-You-Go: The Effects of Integrating Public Health Into Pension Systems*

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Abstract

This paper develops an overlapping generations model where the public health sector and the pension system are interlinked. We assume that pay-as-you-go (PAYG) pensions partially depend on health status during old age, by introducing a component which is indexed to the average level of disability. Under that modeling, reducing dependency during old age will lower the pension benefit to be distributed, as the need to finance long-term care services will also decrease. We then study the effects of introducing such a 'comprehensive' Social Security system on individual decisions, capital accumulation, and welfare. We first show that under certain conditions, public health investment can boost savings and capital accumulation in the long run. Second, we find that introducing the health-dependent component to the pension benefit allows for welfare improvements. In that sense our analysis highlights an important policy recommendation: making PAYG pension schemes partially health-dependent can be beneficial to society.

JEL Classification: H55, I15, O41.

Keywords: Public Health Investment, PAYG Pension System, Disability, Overlapping Generations, Long-term Care.

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Highlights

- When ‘pay-as-you-go’ pension benefits are partially conditional on the disability/health status of the retired, public health investment becomes crucial.
- Investing in curative health spending to improve the quality of elderly life then also reduces health-dependent pension benefits.
- Anticipating this effect of a health-dependent pension, working individuals will save more for old age if the health tax, financing health investments, is not too high.
- Introducing a health-dependent component in the pension formula can thus improve overall welfare, as capital formation and wages increase alongside saving and health.

1 Introduction

Because of impressive health care improvements over the last fifty years, people in all developed countries live longer. But this rise in longevity does not come without challenges. More than two out of five people aged 65 or older report having some sort of functional limitation – ranging from sensory, physical, mental, or self-care disabilities – so that the importance of long-term care (LTC) has grown together with the number of elderly (Siciliani, 2013). LTC is very costly, however, and many possible solutions have been discussed over the years to guarantee sufficient care.[1]

A first solution would be to develop private LTC insurance products, but this is shown to be ineffective. Individuals often fail to insure themselves against the increasing risk of becoming dependent at a later age, despite the substantial costs associated with the loss of autonomy. Many explanations have been suggested for this 'LTC insurance puzzle', both on the supply-side (e.g. adverse selection leading to high loading costs) and the demand-side (e.g. myopia, bequest motives) of the market.[2] A second solution would be to let the family take care of their elderly, through either formal (in cash) or informal (in time) care (De Donder and Leroux, 2017). But, this may come at the expense of reduced labour supply of informal caregivers, higher psychological costs for the latter, and possibly lower aggregate welfare in society.

The last solution, and this is the one we explore in this paper, is to let the government step in to tackle the increasing financial needs of our dependent elderly. More specifically, we propose a 'comprehensive' Social Security system that combines both a health system and a pension system which are closely interlinked. On the one hand, the government invests in public health. On the other hand, the pension benefits are augmented by a health-related component, indexed to the average level of disability during old age.[3] Reducing dependency during old age will hence lower the pension benefit, as the need to finance long-term care services also decreases. As a result, the government has a double interest in investing in public health. First, better health will increase individual welfare as it improves *quality* of life during old age. Second, better health will decrease financial pressure on the Social Security system. This paper thus aims to show how public health investment may be useful in designing

[1] We follow Cremer (2014) in defining long-term care as: "the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform daily activities such as bathing and preparing meals." LTC is mainly targeted at the elderly, with needs arising from various chronic diseases (mostly diabetes and -increasingly- cancer), Alzheimer or other forms of dementia.

[2] On the LTC insurance puzzle, see among others, Pauly (1990), Pestieau and Ponthière (2012), Brown and Finkelstein (2009), Brown and Finkelstein (2007), Lockwood (2018) and Boyer et al. (2020).

[3] In Québec in 2015 for instance, the provision of an additional benefit to retirees above a given age was discussed, the so-called *assurance autonomie*, in order to cope with potential loss of autonomy at older age (Hébert, 2016). Because of a change in government majority, it was never implemented.

more efficient pension schemes and in turn, in increasing aggregate welfare.

Compared to the usual views on pension systems, our perspective offers a more optimistic and broader take. In the many countries facing the consequences of population aging, public debate tends to disseminate the idea that health improvements are one of the factors behind unsustainable 'pay-as-you-go' (PAYG) pension systems.[4] Conversely, our approach shows that by investing in public health and hence improving the quality of life during old age, the need for additional LTC expenditures is mitigated. We show that this, in turn, can boost saving, nudge up wages and ultimately increase overall welfare in society. This will in particular be the case when the level of health taxation is not too high and the PAYG pension scheme is made sufficiently health-dependent.

Our objective is thus to better understand the effects of increased public health investment on individual behaviour, capital accumulation, growth and welfare when a PAYG public pension scheme partially depends on the old-age average level of disability in society. To do so, we extend the textbook overlapping generations (OLG) model introduced by Diamond (1965) by allowing pension benefits to include both a universal component (the standard PAYG pension benefit) and a component that will be health-dependent (what we call the "disability-augmented" part of the pension benefit). In our view, such a modulation of the pension system is more than relevant as it ties pension benefits to health-status during old age, something which clearly influences the amount of resources needed when old, in particular if individuals become dependent and need LTC.

Since our aim is to study the effects of reducing the *intensity* of dependency during old age, we focus on *curative* health spending. A few examples are building nursing homes, increasing the number of nurses and providing subsidies for LTC expenditures. For this reason we abstract from the fact that (preventive) health care is also likely to boost longevity, and assume that individuals live two periods of fixed length with certainty. Doing so, we hence also abstract from the potential impact of increased longevity on the pension budget constraint. This way, we can exclusively concentrate on the ways in which improving quality of life during old age can affect the economy, regardless of lifespan changes.

In our model, a representative individual derives utility from consumption and from being autonomous during old age. In the first period, he works, contributes to the health system and the Social Security system. He also saves for old-age consumption. In the second period, he consumes the proceeds of his savings and the pension benefit. Public intervention is twofold. First, thanks to health contributions, the government invests in public health by

[4]This is inherent to the design of PAYG pension systems, where working individuals pay for the pensions of retired individuals. If healthier people live longer there will logically be fewer working contributors for each pensioner, a downward trend which is projected to accelerate Pecchenino and Pollard (2005). See also Cigno and Werding (2007) on increasing age-dependency ratios.

making curative expenditures. Indeed, contributions of workers in a given period are directly invested to increase the health condition of the old in the *same* period. This directly increases the elderly' s utility since they value better health (equivalently higher quality of life) during old age. Second, individuals contribute to a PAYG pension system in the first period and receive a pension benefit in the second period. As already mentioned, we will model an augmented version of the standard pension system by allowing the amount of the pension benefit to vary with the average level of disability in the economy. If health at the old age deteriorates (equivalently, the level of dependency is higher in the society), the elderly receive higher pension benefits so as to cope with extra LTC expenditures.

In order to answer our research questions, we start by assuming that the Social Security tax rates are close to those observed in OECD countries and look at how higher public health spending impacts health, capital accumulation and growth. Second, we study the welfare effects of introducing the 'disability-augmented' pension benefit. To do so, we fix a minimum PAYG pension contribution and find the optimal levels of both the public health tax and of the additional Social Security tax rate which would finance the disability-augmented part of the pension benefit. From this, we quantify the welfare increase resulting from implementing such an augmented pension system in a decentralised setting. Importantly, we relate the optimal tax rate levels and the welfare increase to the individual preference for being healthier during old age (i.e. less disabled).

Our main results are twofold. First, we prove analytically and illustrate it numerically that, assuming Social Security contribution rates close to those observed in OECD countries, higher public health expenditures foster capital accumulation and growth, as long as the contribution rate to the health system is not too large. This is the result of two opposing effects. On the one hand, increasing the health tax increases average health (i.e. decreases average disability) in society which decreases pension benefits in the second period, and thus increases the willingness to save for old age. On the other hand, a higher health tax decreases disposable income and thus, savings. If the health tax is not too big, the first effect dominates the second one, savings increase, and so does capital and output per capita. This in turn boosts wages and brings about even more health investment and savings, leading again to higher wages until a new steady state is reached.

The second contribution we make concerns the welfare effects of introducing a health-augmented pension benefit. We show that given the existing PAYG pension system, introducing the disability-augmented pension component in the pension benefit enables to augment individuals' welfare. We also show that the higher the individual preference for health, the larger the welfare gains of augmenting the PAYG system.

The paper proceeds as follows. Section 2 relates our paper to the existing literature.

Section 3 presents the model, and establishes the equilibrium. Section 4 describes the effect of a rise in health taxation on steady-state capital accumulation. Section 5 combines our findings to shed light on the potential welfare ramifications brought about by the kind of mixed pension system we propose. Section 6 concludes.

2 Related Literature

Our paper can be related to at least two strands on the literature. First, it can be related to the vast literature on economic growth (see de la Croix and Michel (2002)) and on overlapping generations (OLG) models which have been building on the seminal work of Diamond (1965). OLG models are particularly suited to study the impact of major demographic changes over consumption, capital accumulation and growth as well as the effect of policy changes on these economic variables. In particular, Diamond's model has been extended to capture the effects of increased longevity, decreasing fertility and human capital formation on pensions (See e.g. Kaganovich and Meier (2012), Fanti and Gori (2012), Cipriani (2014), and Chen (2018)). Interestingly for our paper, Chakraborty (2004) extends the model of Diamond (1965) to allow for public health expenditures to endogenously determine longevity. He shows that raising taxes to finance public health investments then improves survival chances of the elderly. Anticipating a longer lifespan, agents choose to save more to smooth consumption at an older age, thereby boosting capital accumulation in the long run. Our paper differs from the strand of the literature which followed Chakraborty (2004) and assumed endogenous longevity, in at least to respects.[5] First, unlike this literature, we assume that (curative) public health expenditures affect the *quality* of life during old age, or equivalently the intensity of disability, but not the *length* of the retirement period itself or the individual's survival probability. The question we ask is to what extent the reduction in morbidity (due to higher health expenditures) fosters capital accumulation and growth. Second, we look at the interplay between public health programs and PAYG pension systems and, show how their joint design affects capital accumulation, growth and ultimately aggregate welfare, something which is absent from the existing OLG literature.[6]

Second, our paper can be related to the growing literature on rising LTC needs as a consequence of aging, and its impact on social welfare. To date, most of the existing literature has concentrated on the reasons why there exists a long term care insurance puzzle (see

[5]See for instance, Blackburn and Cipriani (2002), Bhattacharya and Qiao (2007), de la Croix and Ponthiere (2010), Jouvet et al. (2010), de la Croix and Licandro (2013), Fanti and Gori (2014), Zhao (2014) and Davila and Leroux (2015) which assume endogenous lifetime.

[6]One exception is Grossmann and Strulik (2017) which studies the effect of medical progress on health spending, public savings for old age and the retirement age. They show that increasing medical progress is in conflict with reducing health inequalities.Contrary to us, they have no private savings and the pension system does not include the disability component.

footnote 2 above). Most of the theoretical papers which rationalize the reasons underlying the absence of LTC insurance are partial equilibrium models.[7] The theoretical literature studying the general equilibrium impacts of rising LTC needs on capital accumulation and growth is still quite scarce. A first exception is Hemmi et al. (2007) which shows how after-retirement health shocks affect the level of precautionary savings and how multiple steady-state equilibria may exist depending on the level of economic development. They assume away any Social Security program. Kopecky and Koreshkova (2014) also calibrate a life-cycle model where individuals face risk on earnings, survival and LTC expenses, but can partially insure through public programs such as Medicaid and Social Security. They show that a sizeable part of wealth is devoted to planning for old-age risks of survival and of needing LTC. They further show that all young generation would benefit from making these public programs more generous. Finally, Canta et al. (2016) study the dynamic of capital accumulation when individuals face a probability to become dependent during old age and when the existence of family norms can influence the level of informal care provided by the family. They allow for a social insurance over the LTC risk but take it as given.

Contrary to these papers, we model a public pension system which is closely linked to the level of public health investment, and more specifically, which allows to account for the fact that if the health at old age deteriorates (resp. improves), this should be reflected in the level of the pension benefits to be distributed. As such, we claim that both public programs should not be considered in isolation, contrary to what is mostly assumed in the above cited literature. Furthermore, we provide a normative analysis by showing that introducing a health component in the pension benefit formula is welfare improving.

Shedding some light on the long-term implications of combining extended health care with conditional pensions, we aim to cover several blind spots in the policy debate as well.

3 The Model

We consider a closed economy populated with perfectly foresighted and identical individuals whose finite lifespan is divided into two periods: youth (the working period), and old age (the retirement period). We assume no uncertainty on lifespan and each period is normalised to one. Also, in the first period of their life, individuals have perfect health, while in the second period, their health has deteriorated (we will come back to this point below).

At every date t, a continuum of identical agents are born. During each time period t the newly born generation of N_t individuals overlaps with the previous one, growing at an exogenous rate of $n \in (-1; +\infty)$, so that $N_t = (1 + n)N_{t-1}$.

[7] See e.g. Pestieau and Sato (2008),Canta and Pestieau (2013), De Donder and Leroux (2014), De Donder and Leroux (2017) and De Donder and Pestieau (2017).

When young, the representative agent born at t inelastically supplies one unit of labor, and earns the competitive wage rate w_t. When retiring, agents consume their accumulated savings as well as the pension benefits (which we formally define below) provided by the government.

The public system consists of two separate pillars: a pension system and a health care system. These two pillars are detailed below.

3.1 Public Health Investment

As mentioned above, individuals have perfect health (normalised to 1) in the first period of their life while their health has deteriorated in the second period. Their health status during old age, or equivalently their level of autonomy is denoted by d_t, and it is modeled by the following equation

$$d_t(h_t) = \bar{d} + \frac{h_t}{1+h_t}, \tag{1}$$

where $\bar{d} < 1$ is the exogenous part of the health status and h_t is public health investment made at period t so as to improve health of the old, in that *same* period t. In that respect, health expenditures can be considered as being curative (as opposed to preventive).[8]

Absent public health investment, i.e. $h_t = 0$, health status during old age would therefore be imperfect (i.e. lower than 1). Yet, in our model, we assume that the government can invest in health and that the health status d_t is increasing and concave in the amount of public health expenditures.

For simplicity, in the rest of the paper, we assume that $\bar{d} = 0$ so that the above function reduces to

$$d_t(h_t) = \frac{h_t}{1+h_t}. \tag{2}$$

This function then satisfies the following properties: $d(0) = 0$, $\lim_{h\to\infty} d(h) = 1$ and $\lim_{h\to 0} d^{'}(h) = 1$. To guarantee that old-age health status will be strictly positive, we reasonably assume that the government always invests some public resources in health.[9]

In order to finance, public health investments h_t at time t, we assume that the government levies a health tax τ_h on the labour income of the current working generation. For the sake of simplicity, we set the proportional tax on gross wages to be constant over time. Assuming that at each period, the health care budget is balanced, we obtain that total expenditures

[8]Such investments range from building hospitals, setting up new vaccination programs, or quite simply extending existing medical services.

[9]In this paper, we do not explicitly model the government decision to invest in health. If zero health investment was possible then there would be no interest in our model anymore. In addition, $h_t = 0$ would lead to $d_t = \bar{d} = 0$, equivalent to full disability, which seems quite extreme.

should be equal to total revenues, that is

$$h_t = \tau_h w_t. \tag{3}$$

Note that the above modeling of the health status allows to abstract from private health insurance motives, since the health status during old age depends *solely* on public health investment.[10] This also allows us to put aside any moral hazard concerns related to private incentives of the agent to invest in health. Of course, the individual health condition depends on many other factors, such as for example, personal behaviour and lifestyle, private health investments and the environment.[11] However, since the objective of this paper is to understand the effect of public health investment on pension systems, capital accumulation and welfare through its impact on old age disability, we abstract from these considerations.

3.2 Health-Dependent Pensions and Public Spending on the Elderly

The pension system is PAYG so that young agents at period t pay for the pension benefits of the retirees in that same period t. In order to finance the pension system, a proportional contribution, $\tau_{p,t}$ is paid by the workers. This contribution takes the following form:

$$\tau_{p,t} = \tau_0 + \tau_1(1 - d_t) \tag{4}$$

where τ_0 and τ_1 are two exogenous positive constants. The contribution paid by workers is then twofold. The first part τ_0 does not depend on the current health status in society; the second one, $\tau_1(1 - d_t)$ does.[12] This second (health-related) contribution part constitutes the main difference with respect to existing models of retirement and LTC, which always consider retirement and LTC systems as two separate systems, and never condition the taxes to be paid on the current average level of disability (equivalently, of morbidity) in society. The idea here is to make workers pay more whenever the health status of the old in the society deteriorates (i.e. d_t is smaller).[13] For that reason, the contribution rate to the pension system depends on time t, through d_t (or h_t).

[10] More precisely, we do not consider private investments in health and/or insurance motives. Moreover, if we consider that the health status is an indicator of disability and of limitations in activities of daily-living, the literature on the "LTC insurance puzzle" has shown that individuals on their own do not insure privately against LTC risks (on this see Pestieau and Ponthière (2012), Brown and Finkelstein (2009), Brown and Finkelstein (2007) and Boyer et al. (2020)) and that most often the public has to step in.

[11] On the determinants of the health status, see among others, Balia and Jones (2008), Contoyannis and Jones (2004), Ettner (1996), Kaplan et al. (1987).

[12] For simplicity, we assume that the relationship between τ_p and d_t is linear. Assuming otherwise would not qualitatively change our results.

[13] Indeed, this is comparable to the "Journée de la solidarité" (i.e. "solidarity day") which was implemented in France in 2004. This policy measure consisted in suppressing one vacation day so as to finance LTC at the old age and disability programs. This can equivalently be seen as imposing an additional tax based on the average old-age disability level in the society.

Assuming that the pension system is balanced at each time period t, each retiree will receive pension benefit p_t of the following form:

$$p_t = (1+n)(\tau_0 + \tau_1(1-d_t))w_t \tag{5}$$

where the left-hand side accounts for all pension benefits given to retirees and the right-hand side accounts for total contributions to the pension system.

The first part in the above equation, i.e. $(1+n)\tau_0 w_t$, accounts for the *traditional* contributive part of the PAYG pension benefit. On the other hand, the second part of the pension benefit, i.e. $(1+n)\tau_1(1-d_t)w_t$, constitutes the novelty of our approach with respect to standard retirement models. This is what we may call the *'disability-augmented'* part of the pension benefit. Indeed, if health were perfect (i.e. individuals remained autonomous at the old age), $d_t = 1$ and the second part would vanish. In that case, retirees would obtain a lump sum benefit, $p_t = \tau_0(1+n)w_t$, independent from the health status in society. Inversely, whenever health is imperfect, i.e. $d_t < 1$, workers face an additional tax contribution which is redistributed to retirees. The better (resp. the worse) the health condition at old age in society (through higher -resp. lower- public investment in health), the lower (resp. the higher) would be the tax rate and thus the pension benefits to be served. To sum up, this additional benefit, indexed to the average level of disability in society, can then be used to finance the additional LTC expenditures caused by a lower level of autonomy during old age.

3.3 Individuals

The lifetime welfare of perfectly foresighted individuals of generation t is represented by a homothetic and separable utility function U_t, defined over consumption of a private good at the young and the old ages, and over the health status at the old age:

$$U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}) + \gamma\, d_{t+1}, \tag{6}$$

with $c_{1,t}$ and $c_{2,t+1}$, denoting private consumption respectively at the young age and when retired, and d_{t+1} is the health status in old-age which, as we detailed in Section 3.1, depends on public health investment. The parameter $0 < \beta \leq 1$ accounts for time preferences, while $\gamma > 0$ is an individual preference parameter.[14] A higher γ corresponds to higher weight attributed to health (relative to consumption in both periods) during old age, and as we show below, it will motivate higher investment in health and higher health-related tax rates. As a preference for desired health-status during old age, this parameter can in other words be thought of as a health-oriented expression of 'future orientation'.[15]

[14]This modeling of the preference for health is similar to Fonseca et al. (2020) and Hall and Jones (2007).

[15]In psychology and related fields, 'future orientation' is broadly defined as the extent to which an individual thinks about the future, anticipates future consequences, and plans ahead before acting. In a survey covering

Two remarks regarding the modeling of the utility function are in order. First, assuming separability is standard in the OLG and the health economics literature. Second, we implicitly assume here that there is no complementarity or substituability between health (or disability) and consumption in old age, or equivalently that the marginal utility of old-age consumption does not depend on the health status (i.e. cross derivatives between second-period consumption and health are null).[16] This is an important assumption we make in order to keep our analytical part tractable. In addition, and as also argued by Hall and Jones (2007), whether substitutability and complementarity should be modeled is debatable. It crucially depends on the type of consumption goods considered, whether it includes traveling, food or medical expenditures for example, or the adaptation of the house to limitations in daily-life activities. We abstract from this specificity in this paper.[17]

At time t, young individuals join the workforce and offer one unit of labour to firms, and receive the competitive wage w_t. This salary is taxed to finance both public health investment and the pension system. Therefore, the budget constraint of the young agent at time t is given by

$$c_{1,t} + s_t = w_t(1 - \tau_h - \tau_{p,t}). \tag{7}$$

where s_t are the individual's savings. Savings are deposited in a mutual fund accruing at a gross return of r_{t+1}. Note that the overall tax rate needs to be below unity, that is such that $(\tau_{p,t} + \tau_h) \in (0, 1]$.

During old age, consumption c_2 is financed out of savings and pension benefits. The budget constraint of an old agent born at time t then writes as

$$c_{2,t+1} = s_t(1 + r_{t+1}) + p_{t+1}, \tag{8}$$

with p_{t+1} the pension benefit as defined by (5). Substituting equations (5), (7) and (8) into (6) and maximizing the individual's utility function U_t w.r.t. savings, it can be shown that the optimal saving decision s_t of an individual born in period t is equal to

$$s_t = \frac{(1 + r_{t+1})w_t(1 - \tau_h - \tau_{p,t+1})\beta - (1 + n)\tau_{p,t+1}w_{t+1}}{(1 + r_{t+1})(1 + \beta)} \tag{9}$$

where the pension contribution, $\tau_{p,t+1}$ depends on the health status d_{t+1} and thus on h_{t+1} (through equation (2)) over which the individual has no control. Hence, the individual's

French respondents, Apouey (2018) explores this broader notion of time preferences over the future, and finds correlational evidence linking it to preparation for old age (showing for example in saving behaviour, home adaptation and ownership, but *not* in LTC insurance). Similarly to Reckers-Droog et al. (2021), this preference parameter can then be seen as representing the marginal willingness to pay for health, but then specifically oriented towards future old age.

[16] This assumption is similar to Fonseca et al. (2020) and to Hall and Jones (2007).

[17] On the relationship between the utility from consumption and the health status at old age, see De Donder and Leroux (2021).

savings behaviour crucially depends on the size of the pension benefits he expects to receive in old age, which in turn implicitly depend on the level of disability during old age of society, and thus, on public health investments in the next period.

3.4 Firms

At every period t, firms produce a final good whose price is normalised to unity, for an amount $Y_t = F(K_t, L_t)$, that can be consumed, saved or invested in public health. Since in our model, every agent works, $L_t = N_t$ and assuming a standard Cobb Douglas production function, we have:

$$F(K_t, N_t) = AK_t^{\alpha} N_t^{1-\alpha} \tag{10}$$

with $\alpha \in (0, 1)$ and where $A > 0$ accounts for the exogenous technology productivity or equivalently, total factor productivity.

We define the production function in per capita terms:

$$y_t = f(k_t) = Ak_t^{\alpha},$$

with k_t capital per unit of labor. Perfect competition in the goods market implies that both capital and labor are paid at their respective marginal productivity, that is

$$w_t = (1 - \alpha)Ak_t^{\alpha} \qquad \text{and} \qquad R_t \equiv 1 + r_t = \alpha Ak_t^{\alpha-1} \tag{11}$$

where $R_t \equiv 1 + r_t$ is the relationship between the rental rate of capital R_t and the interest rate r_t when capital fully depreciates. Assuming full depreciation of capital, we obtain the following relationship between capital and savings:

$$k_{t+1} = \frac{s_t N_t}{N_{t+1}}. \tag{12}$$

3.5 Equilibrium

Given the initial capital stock k_0, the competitive equilibria are characterized by a sequence of $\{k_t\}$ that satisfies the the savings condition defined in (9), the full depreciation of capital equation (12), the prices equation (11) and the health status equation (2). A steady state equilibrium is an equilibrium in which the values of k_t (and then of all the other variables) is constant over time. The following proposition provides an existence and a uniqueness result for the steady state equilibria.

Proposition 1. *Suppose that $\tau_h + \tau_0 + \tau_1 < 1$. Then, for a given initial level of capital $k_0 > 0$, the dynamics $\{k_t\}$ of the per capita stocks of capital over time is well defined. The system admits two steady states: the trivial steady state where $k^* = 0$ which is unstable and a positive asymptotically stable steady state $k^* > 0$.*

Proof. See Appendix A. □

For further use, we denote the related steady state values of consumption in both periods and the health status as c_1^*, c_2^* and d^*.

4 Public Health Investment and Capital Accumulation

Having established the equilibrium, we now focus on the long-term welfare effects of public health investment when the pension system is made contingent on the old-age level of disability in society. To do so, we find the impact of exogeneously increasing τ_h on the steady-state level of capital k^*. This is crucial since the level of capital determines long-term outcomes such as wages, interest rates, and ultimately welfare.

Such a comparative statics exercise is far from trivial. On the one hand, a higher health tax reduces disposable income, in turn discouraging savings and eroding capital stock. On the other hand, a higher health tax increases health at the old-age (through higher public health investments) so that the contribution rates to finance pensions decrease, which increases disposable income, increases the willingness to save and the accumulation of capital. Nonetheless, Proposition 2 below shows that, if the health-related pension contribution rate, τ_1 is sufficiently high, the latter effect dominates. This will be true also as far as the health taxation rate τ_h is not too high and below some threshold $\bar{\tau}_h \in [0, 1]$ (defined in Appendix A) which itself depends on the specific values of τ_0, τ_1.

Proposition 2. *In an economy where the PAYG pension benefits depend on the average societal health status during old age, if τ_1 is sufficiently high, an increase in public health investment through an increase in τ_h boosts the steady state stock of capital, k^* if the health tax rate remains below some threshold $\bar{\tau}_h \in [0, 1]$.*

Proof. See Appendix A. □

When health taxation is low, i.e. $\tau_h < \bar{\tau}_h$, we find that the steady state level of capital increases following an increase in τ_h. In that situation, the increase in the health tax rate decreases the pension contribution rate (this corresponds to the second part in eq. 4) which in turn increases disposable income and the willingness to save for old age. This effect is found to dominate the direct negative impact of a higher τ_h on disposable income of the young. The overall effect is an increase in savings and thus, in capital accumulation k^*. This, in turn, results in higher wages, and into even higher health investment and a better health condition of the elderly. This sparks off an indirect general equilibrium feedback effect which further encourages saving, and serves as a catalyst to accumulate capital. As a result, the steady-state output per worker increases.[18]

However, this multiplier effect is only triggered under specific conditions. First, the health tax cannot be too high, i.e. $\tau_h < \bar{\tau}_h$. Above this threshold, the agent may be liquidity constrained and may not be able to increase further his savings. Second, the contribution rate $\tau_{p,t}$ must be sufficiently responsive to the level of disability in old age d_t, that is τ_1 must be sufficiently high. This second point is quite intuitive, as our main mechanism works through the health-dependent contribution to the pension system. If an increase in τ_h is not allowed to substantially reduce the health-dependant contribution rate (through higher d_t), the increase in τ_h will rather inflate the overall (health plus pension) tax burden, so that disposable income is likely to decrease, together with savings and capital accumulation.

To illustrate how the steady state level of capital responds to an increase in the health tax rate, we perform a very simple numerical analysis in Table 1. To this end, we set the parameter values at the following levels. First, we assume that each representative young adult individual (whose life starts at 20 years old) lives two periods of 40 years each. Assuming an annual discount rate equal to 1.25%, we find that $\beta = 0.6$, which is similar to the value of β in Žamac (2007).

We use a capital-output elasticity of $\alpha = 0.33$ as in de la Croix and Michel (2002). We then set the contribution rate at $\tau_0 = 8\%$ and $\tau_1 = 7.5\%$, because a majority of the OECD countries have rates lying in this range.[19] Following de la Croix and Michel (2002), we consider the replacement rate in a single-parent model over a period of 40 years, so that $(1+0.025)^{40} = 1+n$, which gives an exogenous population growth rate of $n = 1.685$. Finally, we set the scale parameter A equal to 100.[20] We then perform a numerical analysis considering a range of health tax rate τ_h between 0% to 20%.
When τ_h is set to zero, the corresponding steady state level of capital is given by $k^* = 16.7470$. Increasing the health tax, we find that the steady state level of capital increases up to 19.7029 when $\tau_h = 2\%$. Yet, for values of the health tax rate larger than the threshold $\bar{\tau}_h \simeq 2.3\%$, we find that increasing further public health taxation reduces capital accumulation. Table 1 then confirms the theoretical results we find in Proposition 2.

All in all, this section shows that the observed positive relationship between increased public health expenditures and capital accumulation (when τ_h is not too high) is driven by

[18] A similar mechanism where health investment bears on economic growth can be found in Chakraborty (2004), Fanti and Gori (2011) and Fanti and Gori (2014).

[19] See for instance the graphs available at `https://data.oecd.org/tax/social-security-contributions.htm`. Over the period 2000-2020, the average social security contribution rates in the OECD countries have been around 8%, with France exhibiting rates between 14 and 16%.

[20] This level is admittedly a bit high. For instance, Chakraborty (2004) uses levels of 25 and 50. But, since this is a scale parameter, we decided to leave as it is.

Table 1: The effect of a positive health shock

τ_h	h^*	k^*	Effect on k^*
0%	0.000	16.7470	N/A
1%	1.7806	19.3354	positive
2%	3.5834	19.7029	positive
3%	5.3727	19.6756	negative
5%	8.8956	19.2862	negative
10%	17.3461	17.8608	negative
15%	25.2588	16.3256	negative
20%	32.6042	14.7982	negative

the young individuals anticipating a better health status in old age and thus lower public pensions. Obviously, this would not occur under a standard (non-augmented) PAYG pension system. When $\tau_1 \to 0$, there is no impact of increased health expenditures on the amount of pension benefit received and thus, increasing public health taxation always reduces capital accumulation.

Note finally that if we were to consider the effect of τ_1 on the steady state level of capital we would instead observe a monotonic behavior: increasing τ_1 would always decrease capital accumulation k^* at the steady state. To understand this, it is sufficient to note that savings and pension benefits are substitutes in transferring resources from the young age to the old age period. Increasing τ_1 (or equivalently τ_0) would unambiguously increase p_t which in turn would decrease the willingness to save for old age. In addition, higher first-period taxation would decrease disposable income and as such the saving capacity. These two effects go in the same direction as to decreasing savings and thus capital accumulation in equilibrium.

5 Welfare

In this section, we first derive the optimal tax rates τ_1 and τ_h, taking τ_0 as given. We do so because in reality and for different historical and economic reasons (e.g. insurance motive, myopia) which we do not model here, public pension systems do exist. As such, because we start from a situation where a standard PAYG system already exists (i.e. $\tau_0 > 0$), we consider a *second-best* problem and show to what extent the pension scheme should be augmented to take into account the disability-related component. Second, we quantify the welfare increase associated to augmenting the standard pension system.

5.1 Optimal taxation

In this section, we derive the second-best tax rate levels, τ_1^* and τ_h^*, postulating the existence of a pension system that would provide benefits during old age to the individual (as it is the case in most developed countries). To do so, we exogenously fix $\tau_0 \geq 0$ and we find the values of τ_1 and τ_h that would maximize the utility of the representative agent at the steady state when varying the health preference parameter γ. More precisely, we study the impact of varying γ on the (non-trivial) steady state equilibrium characterized in Proposition 1.

Under our general equilibrium framework, choosing the optimal values for τ_1 and τ_h consists in maximising the utility of the representative agent, eq. (6), taking into account that c_1^*, c_2^* and d^* depend both directly on these tax rates as well as indirectly on them. Indeed, c_1^*, c_2^* and d^* (or h^*) depend on τ_1 and τ_h also through k^* which in turn determines wages and pension benefits. Deriving analytically the first-order conditions of this problem with respect to τ_1 and τ_h proves to be tedious and would not provide interesting insights since we would obtain many different effects going in opposite directions.[21] This is why in the following we resort to numerical simulations and show how the optimal tax rates τ_1^* and τ_h^* vary with the preference parameter γ. We also consider different possible values of τ_0 ranging from 0% to 15%.[22] Our results are reported in Table 2.

Table 2: The effect of γ on tax rates

	$\tau_0 = 0\%$			$\tau_0 = 8\%$		
	τ_1^*	τ_h^*	$\tau_1^*(1-d)$	τ_1^*	τ_h^*	$\tau_1^*(1-d)$
$\gamma = 1$	0%	4.6%	0%	0%	4.5%	0%
$\gamma = 2$	0%	6.6%	0%	0%	6.5%	0%
$\gamma = 3$	0.9%	8.1%	0.1%	0%	8.0%	0%
$\gamma = 4$	3.0%	9.4%	0.1%	0%	9.2%	0%
$\gamma = 5$	4.7%	10.5%	0.2%	2.0%	10.3%	0.1%
$\gamma = 6$	6.1%	11.5%	0.2%	3.6%	11.3%	0.2%
$\gamma = 7$	7.3%	12.4%	0.3%	5.0%	12.2%	0.2%
$\gamma = 8$	8,3%	13.2%	0.3%	6.2%	13.0%	0.2%
$\gamma = 9$	9.3%	14.0%	0.4%	7.3%	13.8%	0.3%
$\gamma = 10$	10.1%	14.7%	0.4%	8.3%	14.5%	0.3%
	$\tau_0 = 5\%$			$\tau_0 = 15\%$		
	τ_1^*	τ_h^*	$\tau_1^*(1-d)$	τ_1^*	τ_h^*	$\tau_1^*(1-d)$
$\gamma = 1$	0%	4.5%	0%	0%	4.5%	0%
$\gamma = 2$	0%	6.5%	0%	0%	6.4%	0%
$\gamma = 3$	0%	8.0%	0%	0%	7.9%	0%
$\gamma = 4$	1.0%	9.2%	0.1%	0%	9.1%	0%
$\gamma = 5$	3.0%	10.4%	0.1%	0%	10.1%	0%
$\gamma = 6$	4.5%	11.4%	0.2%	1.7%	11.1%	0.1%
$\gamma = 7$	5.8%	12.2%	0.3%	3.2%	12.0%	0.2%
$\gamma = 8$	7.0%	13.1%	0.3%	4.5%	12.8%	0.2%
$\gamma = 9$	8.0%	13.8%	0.3%	5.7%	13.5%	0.3%
$\gamma = 10$	8.8%	14.6%	0.3%	6.7%	14.2%	0.3%

[21] This would become even more complex when deriving comparative statics with respect to γ.

[22] We use the same parametrization as in the previous section: $\alpha = 0.33$, $\beta = 0.6$, $A = 100$ and $n = 1.685$. In unreported simulations, we also find that the level of health d^* is always comprised between 0.90 and 0.96.

First note that the level of the contribution to the health system, τ_h^*, is always strictly positive, whatever the levels of τ_0 and τ_1^*. This is directly related to the fact that individuals value health in their utility function so that $h^* > 0$ is always optimal. On the contrary, τ_1^* becomes positive only above some γ threshold which will depend positively on the level of τ_0.[23] This is due to the fact that both τ_0 and τ_1 put pressure on financial resources available to the individual in the first period, so that when τ_0 increases, the minimum γ level for τ_1^* to be positive must also increase.

Let us now concentrate on the role of $\gamma > 0$ on the optimal pension design. An increase in the health-preference parameter always increases the optimal levels of the tax rates τ_1 and τ_h, for any given level of τ_0.[24] Indeed, when people exhibit higher preference for their health during old age, it is optimal for a government who seeks to maximise welfare to increase health-related taxes. Therefore, the optimal augmented-pension system crucially depends on the agents' relative preference for health during old age and the higher this preference, the higher should the health component in the disability-augmented pension system be.

Let us further relate the above findings with existing policies and formulate some policy recommendations. It is reasonable to assume that in the recent years, individuals' preference for health in general and for health during old age have increased as a consequence of first, population aging (and with it, an increasing probability to become dependent) and second, of our economies becoming richer and more informed of health issues. The above results then demonstrate how accurate it would be to implement such a health-augmented pension system in this context.

5.2 Quantification of welfare

We now concentrate on quantifying the welfare effect of implementing an augmented pension system, that is the welfare effect of allowing the pension benefits to be conditioned on the average disability level in society. To do so, we set $\tau_0 = 8\%$ as in the average of OECD countries and we compare the welfare of the representative individual in steady state when $\tau_1 = 0$ (that is, when the pension system is a "traditional" one) with its welfare when $\tau_1^* > 0$ is set at its optimal steady state level (i.e. the pension benefit is "disability-augmented"). In both cases, τ_h is chosen optimally (i.e. as a solution of the individual's problem). We then quantify by how much society's welfare is increased when augmenting the standard PAYG pension system by its disability-related component. As we showed in the previous section, whether the optimal level of τ_1^* is strictly positive depends on the size of the health-preference parameter. In order to be able to compare welfare in the two pension situations (whether it

[23] Equivalently, the minimum γ level at which τ_1^* becomes positive increases with the level of τ_0.

[24] Note that the optimal tax rates increase nonetheless at a lower rate when γ becomes higher, which is due to the concavity with respect to consumption of the utility function.

is augmented or not), we will then concentrate on values of γ between 5 and 10 such that $\tau_1^* > 0$.

Our results are reported in Table 3, which is divided in three parts. In the first four columns, we report the values of c_1^*, c_2^*, h^* (and τ_h^*) when τ_0 is fixed to 8%, τ_1 is exogenously fixed to 0% but τ_h maximises the steady state utility of the representative agent. In the second part of the table (columns five, six, seven and eight), we report the values of c_1^*, c_2^*, h^* (and τ_h^*) when τ_0 is still fixed to 8% but both τ_1 and τ_h are (jointly) chosen to maximize the utility of the representative agent at the steady state.[25] The last column reports the relative variation in the utility between a standard PAYG system ($\tau_1 = 0\%$) and the augmented PAYG system ($\tau_1^* > 0$).

Table 3: The welfare effect of augmenting the PAYG system when $\tau_0 = 8\%$

	$\tau_1 = 0\%$				$\tau_1^* > 0$				% variation in U
	c_1	c_2	h	τ_h^*	c_1	c_2	h	τ_h^*	
$\gamma = 5$	87.6	297.5	0.947	10.2%	88.5	291.9	0.947	10.3%	+0.002%
$\gamma = 6$	85.5	299.0	0.950	11.1%	87.0	288.8	0.951	11.3%	+0.005%
$\gamma = 7$	83.6	300.5	0.954	11.9%	85.7	286.2	0.954	12.2%	+0.009%
$\gamma = 8$	81.9	301.8	0.956	12.7%	84.5	284.0	0.957	13.0%	+0.014%
$\gamma = 9$	80.3	303.0	0.958	13.4%	83.4	282.0	0.959	13.8%	+0.018%
$\gamma = 10$	78.8	304.1	0.960	14.0%	82.3	280.2	0.961	14.5%	+0.022%

Table 3 clearly shows that, given the existence of an universal PAYG pension system, augmenting the system with a health-dependent pension increases welfare when τ_1 and τ_h are set at their optimal levels and, provided that γ is sufficiently high. Note that the effects of augmenting the pension system may look small when considering variations in utility, but this is due to the concavity of the log-utility function. For large values of c, the utility function is not very sensible to variations of c. Still, we can observe a relevant change in the consumption behavior of the agents.

6 Concluding Remarks

This paper has shown that a Social Security System which combines both a public health system and an augmented-pension system, accounting for disability during old age, can be beneficial to society. Our model clearly departs from previous contributions by proposing the implementation of a PAYG pension scheme in which pension benefits are augmented by a health-related component, indexed to the average level of disability during old age.

Assuming an overlapping generations framework, our analysis produces several insightful predictions regarding the effects of Social Security taxation on individual decisions, capital

[25] We do not report the values of τ_1^* here, but these correspond to the values reported in the part of Table 2 where $\tau_0 = 8\%$.

accumulation, and welfare. First, an increase in public health investment through an increase in the tax rate can boost the steady state stock of capital. This result emerges because health taxation decreases average disability in society which in turn decreases pension benefits in the second period, and thus increases the willingness to save for old age. Second, given the existing PAYG pension system, introducing the disability-augmented pension component in the pension benefit allows for welfare improvements for the elderly. Third, the higher the individual preference for health in the society, the larger the welfare gains of augmenting the PAYG system.

As LTC expenditures are projected to rise dramatically in the near future, our model lays bare the importance of public health investment against disability. As we showed, introducing a health-dependent pension scheme could be part of the solution and, extending the standard PAYG pension benefit to allow for a long-term care dimension has been proven to be beneficial for society.

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A Proofs

Proof of Proposition 1. Using (9) and (11) one can rewrite (12) as

$$k_{t+1}(1+n) = \frac{(\alpha A k_{t+1}^{\alpha-1})(1-\alpha)Ak_t^{\alpha}(1-\tau_h-\tau_{p,t+1})\beta-(1+n)\tau_{p,t+1}(1-\alpha)Ak_{t+1}^{\alpha}}{(\alpha A k_{t+1}^{\alpha-1})(1+\beta)}$$

where $\tau_{p,t+1}$ is defined by (4). Replacing further for equations (2) and (3), we obtain after some manipulations, that

$$k_{t+1}(1+n)(1+\beta)+\frac{(1-\alpha)k_{t+1}(1+n)(\tau_0+\tau_1+Ak_{t+1}^{\alpha}\tau_0\tau_h(1-\alpha))}{\alpha+Ak_{t+1}^{\alpha}\tau_h(1-\alpha)\alpha k_{t+1}^{\alpha}}$$
$$=(1-\alpha)\left(Ak_t^{\alpha}\beta-\frac{Ak_t^{\alpha}(\tau_0+\tau_1+\tau_h+Ak_t^{\alpha}\tau_h(\tau_0+\tau_h)(1-\alpha))\beta}{1+Ak_t^{\alpha}\tau_h(1-\alpha)}\right). \quad (13)$$

Denoting by $G(k_{t+1})$ (respectively, $F(k_t)$) the left- (resp. right-) hand side of the above condition, we have that

$$G(k_{t+1}) = F(k_t). \quad (14)$$

It can be proven (see the Appendix) that for any $k_t > 0$, there exists a unique $k_{t+1} > 0$ satisfying equation (14) so that given an initial level of capital $k_0 > 0$, equation (14) uniquely defines a sequence of $\{k_t\}$ of positive per capita stocks of capital over time. The values of k_t in steady state correspond to the k^* satisfying $G(k^*) = F(k^*)$.

F and G are two functions that map $\mathbb{R}_+$ in $\mathbb{R}_+$ (it is obvious for G, we need the hypothesis $\tau_1+\tau_0+\tau_h<1$). Moreover $G(0)=0$ and, if we compute its derivative, we can easily see that $G'>0$ and $\lim_{k\to+\infty} G(k)=+\infty$ so, for any choice of k_t there exists a unique k_{t+1} which satisfies the equation and so (14) defines, given k_0, a (unique) sequence k_t.

We can also observe that $F(0)=G(0)=0$ (so in particular k^* is a steady state), that (if $1-\tau_h-\tau_1-\tau_0>0$) $\lim_{k\to 0} F'(k)=+\infty$, $\lim_{k\to 0} G'(k)=\frac{\alpha(1+\beta)+(1+n)(\tau_0+\tau_1)(1-\alpha)}{\beta\alpha}\in(0,+\infty)$, that $\lim_{k\to+\infty} F'(k)=0$, $\lim_{k\to+\infty} G'(k)=\frac{\alpha(1+\beta)+(1+n)(\tau_0+\tau_1)(1-\alpha)(1-d_1)}{\beta\alpha}\in(0,+\infty)$ so $F(k)>G(k)$ for small k (different from 0) and $F(k)<G(k)$ for large values of k so there exists at least a solution of $F(k)=G(k)$ i.e. a steady state. To prove the uniqueness of strictly positive steady state one can observe that, after some computation, the condition $F(k)=G(k)$ for $k>0$ is equivalent to

$$\frac{(1+n)}{\beta A}k^{1-\alpha}=\frac{\left[(1-\tau_0-\tau_1-\tau_h)+Ak^{\alpha}\tau_h(1-\tau_0-\tau_h)\right]\alpha(1-\alpha)}{\tau_0+\tau_1+\alpha(1+\beta-\tau_0-\tau_1)+Ak^{\alpha}\tau_h(1-\alpha)(\tau_0(1-\alpha)+\alpha(1+\beta))}$$

after taking the power ($\frac{1}{1-\alpha}$ of both side of this expression one can verify that the function on the left hand side is linear while the function on the right hand side (after some computation) is concave so the strictly positive steady state is unique. We can denote it by k^*. Since

$G(k) > F(k)$ for all $k > k^*$ and $G(k) < F(k)$ for all $0 < k < k^*$ then k^* is globally asymptotically stable (and the null steady state is unstable).

□

Proof of Proposition 2. The unique strictly positive steady state k (see Proposition 1) is defined by the equation $G(k) = F(k)$ or $0 = H(k)$ with $H(k) := G(k) - F(k)$. Using the implicit function theorem, we have that $\frac{dk}{d\tau_h} = -\left(\frac{\partial H}{\partial k}\right)^{-1}\left(\frac{\partial H}{\partial \tau_h}\right)$. The latter can be computed explicitly and we obtain:

$$\frac{dk}{d\tau_h} = -(1-\alpha)^2 A k^{1+\alpha}\frac{I_1}{I_2}$$

where

$$I_1 = -k(1+n)\tau_1(1-\alpha) - \alpha\beta - Ak^{\alpha}(\tau_1 - 2\tau_h)(1-\alpha)\alpha\beta + A^2k^{2\alpha}\tau_h^2(1-\alpha)^2\alpha\beta$$

and

$$\begin{aligned} I_2 = (1+n)(1-\alpha)\left[-1 + Ak^{\alpha}\tau_h(1-\alpha)\right]^2 \alpha(1+\beta)k + k(1+n)(\tau_0+\tau_1) \\ + Ak^{1+\alpha}(1+n)\tau_h(1-\alpha)(2\tau_0 + (1-\alpha)\tau_1) + A^2k^{1+2\alpha}(1+n)\tau_0\tau_h^2(1-\alpha)^2 \\ - Ak^{\alpha}(1-\tau_0-\tau_1-\tau_h)\alpha^2\beta - 2A^2k^{2\alpha}\tau_h(1-\tau_0-\tau_h)(1-\alpha)\alpha^2\beta \\ - A^3k^{3\alpha}\tau_h^2(1-\tau_0-\tau_h)(1-\alpha)^2\alpha^2\beta \end{aligned} \quad (15)$$

When τ_1 is high enough (observe that, once other parameters are fixed, k is a function of τ_1 and τ_h is uniformly bounded), one can easily verify that the expression of $\frac{dk}{d\tau_h}$ is strictly positive for $\tau_h = 0$. By continuity, this should also be the case for small values of τ_h. So in particular there exists $\bar{\tau}_h$ such that, once we fix A, α, ρ, β, n, γ, τ_0 and τ_1, k^* is an increasing function of τ_h on $[0, \bar{\tau}_h]$. This threshold level $\bar{\tau}_h$ is implicitly defined by

$$\frac{dk}{d\tau_h} = -(1-\alpha)^2 A k^{1+\alpha}\frac{I_1}{I_2} = 0$$

or equivalently by setting $I_1 = 0$. □